VISCOUS DISSIPATION DURING HEAT TRANSFER IN A PIPE

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Dissipation of mechanical energy affects in many cases the temperature regime in flowing highly viscous materials. We examined this effect for a developed streaming of a non-Newtonian fluid with temperature-independent properties in a pipe with a constant heat flux through its wall.

The principle of the exact solution of a temperature field by separation of variables for the case of constant heat flux through the wall, which leads to a series of eigenfunctions, has been already developed and some results for Newtonian fluids have been described *e.g.* in a monograph by Petukhov[1]. Problems on heat transfer into non-Newtonian fluids through the wall at these boundary conditions have not been solved to a satisfactory extent.

In our previous work[2] we gave a detailed description of the heat transfer problem with dissipation for a non-Newtonian fluid in a pipe at the boundary condition of the first kind – *i.e.* with a constant temperature of the wall. Here we are going to outline the exact solution obtained in an analogous manner for the case of the boundary condition of the second kind – constant heat flux through the wall. As the main contribution of this work, however, we consider the derivation of a simple asymptotic formula for the thermal entrance region, in which most technically important heat transfer processes proceed.

Exact Solution

For a steady-state pipe flow of a non-Newtonian fluid obeying the temperature-independent power-law rheological model

$$\tau = KD^n\,, \tag{1}$$

the heat balance is represented by the equation

$$\varrho c_p U \frac{3+s}{1+s}\left[1-\left(\frac{R}{R_1}\right)^{1+s}\right]\frac{\partial T}{\partial X} = \frac{k}{R}\frac{\partial}{\partial R}\left(R\frac{\partial T}{\partial R}\right) + K\left[\frac{(3+s)\,U}{R_1}\right]^{1+n}\left(\frac{R}{R_1}\right)^{1+s}. \tag{2}$$

A constant input temperature, constant heat flux through the wall and symmetry conditions constitute boundary conditions of the second kind:

$$T = T_0 \quad \text{for} \quad x = 0 \tag{3}$$

$$\partial T/\partial R = q_w/k \quad \text{for} \quad R = R_1 \tag{4}$$

$$\partial T/\partial R = 0 \quad \text{for} \quad R = 0\,. \tag{5}$$

Basic features of solution of system (2)–(5) are analogous to those described earlier[2]: after introducing dimensionless variables

$$r = R/R_1\,, \quad z = xk/(\varrho c_p U R_1^2) \qquad (6), (7)$$

and two new variables with the dimension of temperature

$$T_Q = 2q_w R_1/k\,, \quad T_D = 2(3 + s)^n K U^{1+n} R_1^{1-n}/k\,, \qquad (8), (9)$$

the temperature field assumes the form of

$$T = T_0 + T_Q t_2(r, z) + T_D t_{2D}(r, z)\,. \tag{10}$$

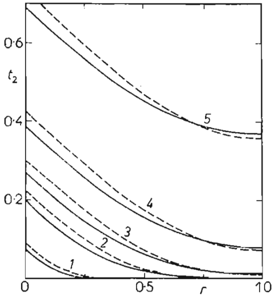

FIG. 1

Function t_2

-------- Newtonian fluid, ——— pseudoplastic fluid with $n = 0.25$. *1* $z = 0.005$, *2* $z = 0.05$, *3* $z = 0.1$, *4* $z = 0.2$, *5* $z = 0.5$.

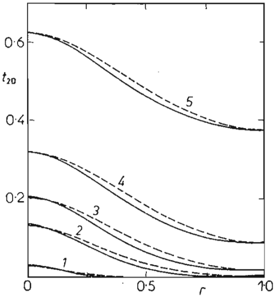

FIG. 2

Function t_{2D}

The notation of the curves is identical with that on Fig. 1.

Two dimensionless functions $t_2(r, z)$, $t_{2D}(r, z)$ can be expressed as series of eingen-functions. Values of first eight eigenfunctions, eigenvalues, and coefficients in these series were calculated numerically and employed further for obtaining values of functions t_2 and t_{2D}. These results are illustrated on Figs 1–2; hence it is obvious that courses of dimensionless functions t_2 and t_{2D}, which were introduced through relations (*8*)–(*10*), do not depend significantly on the non-Newtonian behaviour of the fluid in the region of medium values of z. At large values of z, these functions may be described by asymptotic formulas

$$\lim_{z \to \infty} t_2(r, z) = z + \frac{3 + s}{1 + s} \left[\frac{r^2}{4} - \frac{r^{3+s}}{(3 + s)^2} \right] - \frac{(3 + s)^2 - 8}{8(1 + s)(3 + s)(5 + (s)} , \qquad (11)$$

$$\lim_{z \to \infty} t_{2D}(r, z) = t_2(r, z) - \frac{r^{3+s}}{2(3 + s)} + \frac{1}{2(3 + s) 5 + s)} . \qquad (12)$$

Asymptotic Solution for $z \to 0$

At small values of z, which correspond to most technical applications, the series of eigenfunctions converge slowly and, moreover, during the computation of the functions themselves, problems of numerical stability are encountered. In this region, however, the heat transfer proceeds mainly in an immediate vicinity of the wall, which is also obvious from Figs 1–2, and a similarity solution of the Lévêque type may be applied to this case. This offers a possibility of expressing t_2 as

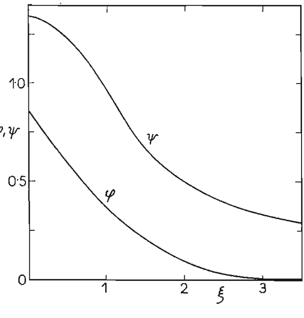

FIG. 3
Functions $\varphi(\xi)$, $\psi(\xi)$

$$t_2 = [z/(3 + s)]^{1/3}\,\varphi(\xi)\,, \tag{13}$$

with

$$\xi = (1 - r)\,[(3 + s)/z]^{1/3}\,, \tag{14}$$

and the function $\varphi(\xi)$ is a solution of the ordinary differential equation

$$3\varphi'' + \varphi'\xi^2 - \varphi\xi = 0 \tag{15}$$

with boundary conditions

$$\varphi'(0) = -\tfrac{1}{2}\,, \quad \varphi(\infty) = 0\,, \qquad (16), (17)$$

which is illustrated on Fig. 3. Similarly, the function t_{2D} can be expressed as

$$t_{2D} = \psi(\xi)\,z^{2/3}(3 + s)^{1/3}/2\,, \tag{18}$$

in which the auxiliary function $\psi(\xi)$ is a solution of the equation

$$3\psi'' + \psi'\xi^2 - 2\psi\xi + 3 = 0 \tag{19}$$

with boundary conditions

$$\psi'(0) = 0\,, \quad \psi(\infty) = 0 \qquad (20), (21)$$

which is illustrated on Fig. 3.

By combining relations (10), (13), and (18) we obtain the approximate formula

$$T(\xi, z) \approx \lim_{z \to 0} T(\xi, z) = T_0 + (2q_wR_1/k)\,[(3 + s)/z]^{1/3}\varphi(\xi) + \\ + (3 + s)^{1/3+n}\,.\,z^{2/3}\,(KU^{1+n}R_1^{1-n}/k)\,\psi(\xi) \tag{22}$$

which yields results satisfactorily accurate for technical calculations up to $z = 0{\cdot}1$.

The whole analysis outlined is based on the assumption of temperature-independent rheological properties of the fluid. In spite of this, similarly as it was shown in the case with constant temperature of the wall[3], we may assume that formula (22) supplemented with correction terms can be employed for real cases with nonisothermal flow, too. It is advantageous to correlate experimental or numerical data by using dimensionless groups from formula (22), from which it also follows that the criterion

$$\Pi_{DO} = z^{1/3}\,KU^{1+n}\,(3 + s)^n/(2q_wR_1^n) \tag{23}$$

can be employed for estimating the effect of dissipation on the temperature field. If it holds $\Pi_{DO} \ll 1$, the dissipation effect can be neglected at $z < 0.1$.

List of Symbols

c_p	specific heat
D	shear rate
k	thermal conductivity
K	consistency coefficient
n	flow index
q_w	heat flux through the wall
r	dimensionless radial coordinate
R	radial coordinate
R_1	radius of the pipe
$s = 1/n$	
T	temperature
T_0	input temperature
T_Q, T_D	characteristic temperatures of the process
t_2, t_{2D}	dimensionless temperatures
U	mean velocity
x	axial coordinate
z	dimensionless axial coordinate
ξ	dimensionless distance from the wall
φ, ψ	dimensionless functions
ϱ	density
Π_{DO}	dissipation criterion in the entrance region

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